PHOENIX FUND INVESTMENT PROJECT

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OPR 620 SESSION 674 -OPERATION RESEARCH I

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Problem Statement

Goal of the Project

Phoenix Fund is establishing a stock investment portfolio for company ABC. which considers invest $4million US dollars only in S&P 500 companies. Phoenix Fund will choose 40 stocks to invest and only consider buying or selling these stocks during the three-year-long investment period. The decision of buying and selling will based on the result of scenario tree with the predication on the economy of the United States and the network optimization to maximize the total revenue generated from the stock investment portfolio. Finally, there will be eight investment portfolios for different conditions and predictions. At the same time, to lower the risk, the stocks to be chosen are divides into two groups: better performance in good economy (positive beta compared to SP500 index-SPY) (Group 1) and better performance in bad economy (negative beta compared to SPY) (Group 2). The observations are extracted from the stock market in 2018. Based on the predictions and results, Phoenix Fund is able to make some decisions for the investment in 2019, 2020, 2021 and 2022 in the U.S. stock market.

Assumptions

The Performances of Stock in Different Conditions[1]: In good economy, the Group 1 stocks that perform better in good economy will increase in the range of 2% to 10%, and the Group 2 stocks that perform worse in good economy will decrease in the range of 1% to 14%. In bad economy, the Group 1 stocks will increase in the range of -1% to 8%, however the Group 2 stocks will decrease in the range of 5% to 11%.
Initial investment: To diversify our portfolio, we limit the number of shares we invest in each stock to be equal or greater than 5 shares.
Transaction Cost: Transaction cost will be charged for each action but with different rates, which is 1.5% of transaction amount for buying and 1.2% for selling.
Selling and Buying: To reduce the prediction error, we limit the number of shares to buy to 150 shares and sell to 100 shares at each period of time, and the cash balance after each investment period to be at least $200.

[1] According to some researches, stocks in some industries are strongly influenced by the economic condition. When the economy is good, those stocks will go up by a large extent, vice versa. However, some stocks in some other industries has a better performance in bad economy so that they can still go up when the economic condition is not pleasant.

Data

To perform the analysis, we collect stock data of 40 companies from S&P list companies from Yahoo Finance. The data we collected is for 4-year period: 2015 to March, 2019 and then we calculate the return for each stock per year. Apart from stock prices, we also collected data of United States GDP for the same period of time as an input for our model.

Methodology

1. Economical Condition Based on Time Series

The GDP records of the United States from 1947 to 2018 have been chosen to analyze the economic conditions, whether it is a bad economical year or a good one (See Appendix 1). And based on the results, the historical stock data are used to predict how would the price of each stock change in a good/bad economy.

2. Clustering Based on Simulated Annealing Algorithm

After knowing the economic conditions, all the 40 stocks are assigned to clusters: In good economy, there are two types of stocks: better-performance ones and worse-performance ones. In bad economy, there are also two types of stocks: better-performance ones and worse-performance ones. To cluster these stocks, first, the stocks are assigned to one cluster. Assume there are 2 clusters (k=2), create a random assignment for the cluster. If the point in the cluster then it will be assigned the value of 1, otherwise 0. By calculating the current distance with the randomly assigned points and comparing with the neighbor, a better cluster there will be. The smaller distances we can get, the better the clustering we will have. Therefore, the fitness model is based on the minimized distance. Whenever each point is assigned to one cluster, the distance of these point to the center of the cluster it is assigned can be calculated. Finally, after iterations, we will know which is the smallest distance that we get from the results. Thus, the assignments of the points that can get the smallest distance is the fittest solution. Based on the clustering result, how these stocks will perform is decided based on the history of the stocks in either good economy or bad economy. See Appendix 2 for cluster result.

3. Scenario Tree for Optimization

The decision of buying and selling the stocks depends on the result from the scenario tree. Each node represents a combination with an actual/predicted economic condition and a specific investment portfolio. The prediction of the U.S. economy condition depends on economic indicators and economic reviews. Based on the reviews, a cut-off point will be decided to help for future decision making. If the value of the indicator is above the cut-off, the economy is predicted to go up, vice versa. The number of shares of stock to sell or buy depends on the linear programming optimization.

General Model

Objective Function:

$$Maximize: \sum \left(X_j^{n-1} + B_j^{n-1} - S_j^{n-1}\right) * P_j^n + C_n$$

Decision Variables:

j = 1, 2, …, m---- number of stocks

i = 0, 1, …, n---- number of investment period

Model: m=40, n=3

$X_j^{n-1} = the\ Number\ of\ Shares\ of\ the\ Stock\ j\ in\ the\ Time\ Period\ n-1$

$B_j^{n-1} = the\ Number\ of\ Shares\ of\ the\ Stock\ j\ to\ Buy\ in\ the\ Time\ Period\ \ n-1$

$S_j^{n-1} = the\ Number\ of\ Shares\ of\ the\ Stock\ j\ to\ Sell\ in\ the\ Time\ Period\ \ n-1$

P_j^n

$= the\ Price\ of\ Stock\ j\ in\ Time\ Period\ n\ when\ the\ portfolio\ matures$

C_n = *Cash Flow in Time Period n*

Constraints:

$$X_j^{i-1} - S_j^i \ \geq 0$$

$$\sum (X_j^0 * P_j^0) + C_0 = 4{,}000{,}000$$

$$\sum (X_j^{n-2} + B_j^{n-1} - \ S_j^{n-1}) * P_j^n \geq 4{,}000{,}000$$

$$X_j^0 \geq 5$$

$$B_j^i \leq 150$$

$$S_j^i \leq 50$$

$$C^i \geq 200$$

$$X_i^j, B_i^j, S_i^j \geq 0$$

$$X_i^j, B_i^j, S_i^j : integer$$

Implementation

We choose R studio to implement our model. Stock price were generated from specific financial and trading libraries in R, quantmod and TTR. Code for stock price, clustering, and linear programming optimization are as follow:

1) Top 100 Stock Price fetching for past 4 years

```
# Loading the required libraries
library(quantmod) ; library(TTR);
start <- as.Date("2015-01-01")
end <- as.Date("2019-03-01")

# Pulling NSE data from Yahoo finance

company<-
c("AAPL","ABBV","ABT","ACN","AGN","AIG","ALL","AMGN","AMZN","AX
P","BA","BAC","BIIB","BK","BKNG","BLK","BMY","C","CAT","CELG","CHT
R","CL","CMCSA","COF","COP","COST","CSCO","CVS","CVX","DHR","DIS",
"DUK","DWDP","EMR","EXC","F","FB","FDX","FOX","FOXA","GD","GE","G
ILD","GM","GOOG","GOOGL","GS","HAL","HD","HON","IBM","INTC","JNJ",
"JPM","KMI","KO","LLY","LMT","LOW","MA","MCD","MDLZ","MDT","ME
T","MMM","MO","MRK","MS","MSFT","NEE","NFLX","NKE","NVDA","OR
CL","OXY","PEP","PFE","PG","PM","QCOM","RTN","SBUX","SLB","SO","SP
G","T","TGT","TXN","UNH","UNP","UPS","USB","UTX","V","VZ","WBA","
WFC","WMT","XOM")
for (i in company) {
symbol = c(i)
print(c(i))
data = getSymbols(symbol,  src = "yahoo", from = start, to =
end,auto.assign = FALSE)
colnames(data) = c("Open","High","Low","Close","Volume","Adjusted")
for(i in data){
data$Stock = rep(c(i))
colnames(data) <-
c("Stock","Open","High","Low","Close","Volume","Adjusted")
}

data = na.omit(data)
closeprice = Cl(data)



# Create the label
data$Return = round(dailyReturn(data$Close, type='arithmetic'),2)
colnames(data) =
c("Stock","Open","High","Low","Close","Volume","Adjusted","Return")

class = character(nrow(data))
```

```
class = ifelse(coredata(data$Return) >= 0,"Up","Down")

data2 = data.frame(data,class)

data2$Stock= symbol
write.csv(data2,file=paste("decision tree
charting_table_",symbol,".csv"))

}
```

2) Clustering

```
# GDP WITH TIME SERIES
 GDP<-read.csv("GDP.csv")
 GDP<-ts(GDP,start=1,frequency=4)
 y<-as.data.frame(GDP)
 y<-y$GDP
 length(y)
 t<-(1:288)
 t2<-t^2
 reg<-lm(y~t+t2)
 summary(reg)
 trendy<-890.81-30.43*t+0.34*t2
 ts.plot(GDP,trendy,col=1:2,ylab="Trend vs. Acutal GDP")
 dt<-GDP-trendy
 plot(dt[,2])
 par(mfrow=c(2,1))
 acf(dt,100)
 pacf(dt,100)
 Eco<-dt[285:288,2]
 dataall<-read.csv("/Users/liuchenyang/Desktop/DATA_ALL_a.csv")
 dataall<-dataall[1:40,]
 y_2015<-dataall[,3]
 y_2018<-dataall[,9]
 change<-round(cbind(y_2015,y_2018),2)
 change<-as.data.frame(change)

 # CLUSTERING STOCKS WITH SIMULATED ANNEALING
 # In Bad Economy
 data<-dataall$Return_a
 data<-scale(data)
 rcur<-runif(40,0,1)
 CurCluster <- ifelse(rcur<0.5,0,1)
 CurCluster <- as.matrix(CurCluster,ncol=1,nrow=40)
 data<-cbind(CurCluster,data)
 C11<-
sum(data[which(data[,1]==1),2])/length(data[which(data[,1]==1),2])
 Center1<-c(C11)
```

```
 C21<-
sum(data[which(data[,1]==0),2])/length(data[which(data[,1]==0),2])
 Center2<-c(C21)
 CurDist<-sum(sum((data[which(data[,1]==1),2]-C11)**2),
          sum((data[which(data[,1]==0),2]-C21)**2))
 BestCluster <- CurCluster
 MinDist <- CurDist

 T<-3
 r<-0.9
 L<-10
 frozen<-0.01

 while (T > frozen) {
  for (i in 1:L) {
   rObs<-runif(1,1,41)
   NeighCluster<-CurCluster
   NeighCluster[rObs,1]<-1-NeighCluster[rObs,1]
   data<-cbind(NeighCluster,df)
   C11<-
sum(data[which(data[,1]==1),2])/length(data[which(data[,1]==1),2])
   Center1<-c(C11)
   C21<-
sum(data[which(data[,1]==0),2])/length(data[which(data[,1]==0),2])
   Center2<-c(C21)
   NeighDist<-sum(sum((data[which(data[,1]==1),2]-C11)**2),
          sum((data[which(data[,1]==0),2]-C21)**2))
   print(NeighDist)
   if (NeighDist < CurDist)
   { CurDist <- NeighDist
   CurCluster <- NeighCluster
   if (NeighDist < MinDist) { MinDist <- NeighDist
   BestCluster <- NeighCluster
   }
   }
   else {
    if (runif(1,0,1) > exp(-(CurDist-NeighDist)/T) ) {
     CurDist <- NeighDist
     CurCluster <- NeighCluster
    }
   }
  }
  T <- r*T
 }
 BestCluster_BAD<-BestCluster
 MinDist_BAD<-MinDist
```

3**) Linear Programming Optimization**

Following code showing solution for stock transaction in Scenario 1, where the economy are always GOOD.
(*Code for Scenario 2 to 8 could be find in the attachment; detail solutions, including variable values and optimal value, for each scenario could be find in Appendix 3.)

```
####################Scenario1######################
library(xlsx)
df <- read.xlsx("Price_8Scenario_new.xlsx", sheetIndex = 1)
price<-df[,4:7]
p0<-round(price$P0,0)
p1<-round(price$P1,0)
p2<-round(price$P2,0)
p3<-round(price$P3,0)


# OPTIMIZATITON:
# Max: P3X0 - 0.015(P1-P0)B1 - 0.012(P1-P0)S1 - 0.015(P2-P1)B2 -
0.012(P2-P1)S2 + C0
X0<-as.vector(p3)
B1<-as.vector(-0.015*(p1-p0))
S1<-as.vector(-0.012*(p1-p0))
B2<-as.vector(-0.015*(p2-p1))
S2<-as.vector(-0.012*(p2-p1))
C0<-as.vector(c(1))
F<-as.vector(c(X0,B1,S1,B2,S2,C0))

# Constratint 1: X0-S1>=0
C11<-diag(40)                          # X0
C12<-matrix(rep(0,40*40),nrow=40,ncol=40)     # B1
C13<--diag(40)                         # S1
C14<-matrix(rep(0,80*40),nrow=40,ncol=80)     # B2,S2
C15<-matrix(rep(0,40),nrow=40,ncol=1)        # C0
C1<-cbind(C11,C12,C13,C14,C15)

# Constraint 2: X1-S2>=0:   X0 + B1 -S2 >=0
C21<-diag(40)                          # X0
C22<-diag(40)                          # B1
C23<-matrix(rep(0,80*40),nrow=40,ncol=80)     # S1,B1
C24<--diag(40)                         # S2
C25<-matrix(rep(0,40),nrow=40,ncol=1)        # C0
C2<-cbind(C21,C22,C23,C24,C25)

# Constraint 3: sum(X0P0) + C = 4000000
C31<-matrix(p0,nrow=1,ncol=40)              # X0
C32<-matrix(rep(0,160),nrow=1,ncol=160)       # B1,B2,S1,S2
C33<-diag(1)                          # C0
C3<-cbind(C31,C32,C33)
```

```
# Constraint 4: P3(X0+B1-S1+B2-S2) >= 5000000
C41<-matrix(p3,nrow=1,ncol=40)             # X0
C42<-matrix(p3,nrow=1,ncol=40)             # B1
C43<--matrix(p3,nrow=1,ncol=40)            # S1
C44<-matrix(p3,nrow=1,ncol=40)             # B2
C45<--matrix(p3,nrow=1,ncol=40)            # S2
C46<-matrix(c(0))                  # C0
C4<-cbind(C41,C42,C43,C44,C45,C46)

# Constraint 5: X0 >= 5
C51<-diag(40)                      # X0
C52<-matrix(rep(0,161*40),nrow=40,ncol=161)   # B1,S1,B2,S2,C0
C5<-cbind(C51,C52)

# Constraint 6: B1 <= 150
C61<-matrix(rep(0,40*40),nrow=40,ncol=40)     # X0
C62<-diag(40)                      # B1
C63<-matrix(rep(0,121*40),nrow=40,ncol=121)   # S1,B2,S2,C0
C6<-cbind(C61,C62,C63)

# Constraint 7: B2 <= 150
C71<-matrix(rep(0,120*40),nrow=40,ncol=120)   # X0,B1,S1
C72<-diag(40)                      # B2
C73<-matrix(rep(0,41*40),nrow=40,ncol=41)     # S2,C0
C7<-cbind(C71,C72,C73)

# Constraint 8: S1 <= 50
C81<-matrix(rep(0,40*80),nrow=40,ncol=80)     # X0,B1
C82<-diag(40)                      # S1
C83<-matrix(rep(0,81*40),nrow=40,ncol=81)     # B2,S2,C0
C8<-cbind(C81,C82,C83)

# Constraint 9: S2 <= 50
C91<-matrix(rep(0,160*40),nrow=40,ncol=160)   # X0,B1,S1,B2
C92<-diag(40)                      # S2
C93<-matrix(rep(0,1*40),nrow=40,ncol=1)       # C0
C9<-cbind(C91,C92,C93)

# Constraint 10: C0 >= 200
C101<-matrix(rep(0,200*1),nrow=1,ncol=200)    # X0,B1,S1,B2,S2
C102<-diag(1)                      # C0
C10<-cbind(C101,C102)

# Constraint 11: C1 >= 200
C111<-matrix(rep(0,40),nrow=1,ncol=40)        # X0
C112<-matrix(-1.015*p1,nrow=1,ncol=40)        # B1
C113<-matrix(0.998*p1,nrow=1,ncol=40)         # S1
C114<-matrix(rep(0,81),nrow=1,ncol=81)        # B2,S2,C0
C11<-cbind(C111,C112,C113,C114)
```

```
# Constraint 12: C2 >= 200
C121<-matrix(rep(0,40),nrow=1,ncol=40)       # X0
C122<-matrix(-1.015*p1,nrow=1,ncol=40)       # B1
C123<-matrix(0.998*p1,nrow=1,ncol=40)        # S1
C124<-matrix(-1.015*p2,nrow=1,ncol=40)       # B2
C125<-matrix(0.998*p2,nrow=1,ncol=40)        # S2
C126<-matrix(c(1))                     # C0
C12<-cbind(C121,C122,C123,C124,C125,C126)

# Constraint 13: B1 >= 0
C131<-matrix(rep(0,40*40),nrow=40,ncol=40)     # X0
C132<-diag(40)                         # B1
C133<-matrix(rep(0,121*40),nrow=40,ncol=121)    # S1,B2,S2,C0
C13<-cbind(C131,C132,C133)

# Constraint 14: B2 >= 0
C141<-matrix(rep(0,120*40),nrow=40,ncol=120)    # X0,B1,S1
C142<-diag(40)                         # B2
C143<-matrix(rep(0,41*40),nrow=40,ncol=41)     # S2,C0
C14<-cbind(C141,C142,C143)

# Constraint 15: S1 >= 0
C151<-matrix(rep(0,40*80),nrow=40,ncol=80)     # X0,B1
C152<-diag(40)                         # S1
C153<-matrix(rep(0,81*40),nrow=40,ncol=81)     # B2,S2,C0
C15<-cbind(C151,C152,C153)

# Constraint 16: S2 >= 0
C161<-matrix(rep(0,160*40),nrow=40,ncol=160)    # X0,B1,S1,B2
C162<-diag(40)                         # S2
C163<-matrix(rep(0,1*40),nrow=40,ncol=1)       # C0
C16<-cbind(C161,C162,C163)

A<-
as.matrix(rbind(C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,
C16))
                 #Cons3  #Cons4          #Cons6     #Cons7     #Cons8
#Cons9  #Cons10-12
b<-
c(rep(0,40),rep(0,40),4000000,4000000,rep(5,40),rep(150,40),rep(15
0,40),rep(50,40),rep(50,40),rep(200,3),rep(0,4*40))
signs<-
c(rep(">=",40),rep(">=",40),"=",">=",rep(">=",40),rep("<=",40),rep("<=
",40),rep("<=",40),rep("<=",40),rep(">=",3),rep(">=",4*40))
res = lpSolve::lp('max',F,A,signs,b)
# Because we take the integer constraint out of the constraint, the
result we have is the upper-bound
upsol<-res$solution
```

```
objup<-res$objval
# Lower bound
low<-round(res$solution[1:200])
x0<-round(res$solution[1:40])
b1<-floor(res$solution[41:80])
s1<-floor(res$solution[81:120])
b2<-floor(res$solution[121:160])
s2<-floor(res$solution[161:200])
C0<-4000000-sum(x0*p0)
lowsol<-c(low,C0)
objlow<-sum(p3*x0)-0.015*sum((p1-p0)*b1)-0.012*sum((p1-p0)*s1)-
0.015*sum((p2-p1)*b2)-0.012*sum((p2-p1)*s2)+C0
#### Solution ####
#Upperbound value of decision variable
upsol
# Lowerbound value of decision variable
lowsol
#Upperbound objective value
objup
# Lowerbound value of objective value
objlow
```

Scenario1 Result:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000
 [10]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000
 [19]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000 62903.412698      5.000000      5.000000
 [28]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000
 [37]      5.000000      5.000000      5.000000      5.000000     45.000000      0.000000      0.000000      0.000000      0.000000
 [46]      0.000000      0.000000      0.000000      4.170156     45.000000      0.000000      0.000000     45.000000      0.000000
 [55]      0.000000      0.000000     45.000000      0.000000      0.000000      0.000000      0.000000      0.000000     45.000000
 [64]      0.000000      0.000000      0.000000      0.000000      0.000000     45.000000      0.000000      0.000000      0.000000
 [73]     45.000000      0.000000      0.000000     45.000000      0.000000      0.000000      0.000000      0.000000      5.000000
 [82]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000
 [91]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000
[100]      5.000000      5.000000      5.000000      5.000000      5.000000     50.000000      5.000000      5.000000      5.000000
[109]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000
[118]      5.000000      5.000000      5.000000      0.000000      0.000000      0.000000      0.000000      8.124791      0.000000
[127]      0.000000      0.000000      0.000000    150.000000      0.000000      0.000000    150.000000    150.000000      0.000000
[136]      0.000000      0.000000      0.000000      0.000000      0.000000      0.000000    150.000000    150.000000      0.000000
[145]      0.000000      0.000000      0.000000      0.000000      0.000000      0.000000      0.000000      0.000000    150.000000
[154]      0.000000      0.000000      0.000000      0.000000      0.000000      0.000000      0.000000     50.000000      5.000000
[163]      5.000000      5.000000      5.000000      5.000000      5.000000      5.000000      9.170156     50.000000      5.000000
[172]      5.000000     50.000000      5.000000      5.000000      5.000000     50.000000      5.000000      5.000000      5.000000
[181]      5.000000      5.000000     50.000000      5.000000     50.000000      5.000000      5.000000      5.000000     50.000000
[190]      5.000000      5.000000      5.000000     50.000000      5.000000      5.000000     50.000000      5.000000      5.000000
[199]      5.000000      5.000000    200.000000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5 62903     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5    45     0
 [43]     0     0     0     0     0     0     4    45     0     0    45     0     0     0    45     0     0     0     0     0    45
 [64]     0     0     0     0     0    45     0     0     0    45     0     0    45     0     0     0     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5    50
[106]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     0     0     0     0     8     0
[127]     0     0     0   150     0     0   150   150     0     0     0     0     0     0     0   150   150     0     0     0     0
[148]     0     0     0     0     0   150     0     0     0     0     0     0     0    50     5     5     5     5     5     5     5
[169]     9    50     5     5    50     5     5     5    50     5     5     5     5     5    50     5    50     5     5     5    50
[190]     5     5     5    50     5     5    50     5     5     5     5   226
> #Upperbound objective value
> objup
[1] 5196053
> # Lowerbound value of objective value
> objlow
[1] 5196044
```

* Since there are 200 integer variables in this problem, including X0, S1, B1, S2 and B2, which increase the difficulty of finding an optimal solution. However, with all the methods we learned, we are not able to find an optimal solution. Hence, we decide to search for the primal bound for the maximization by finding

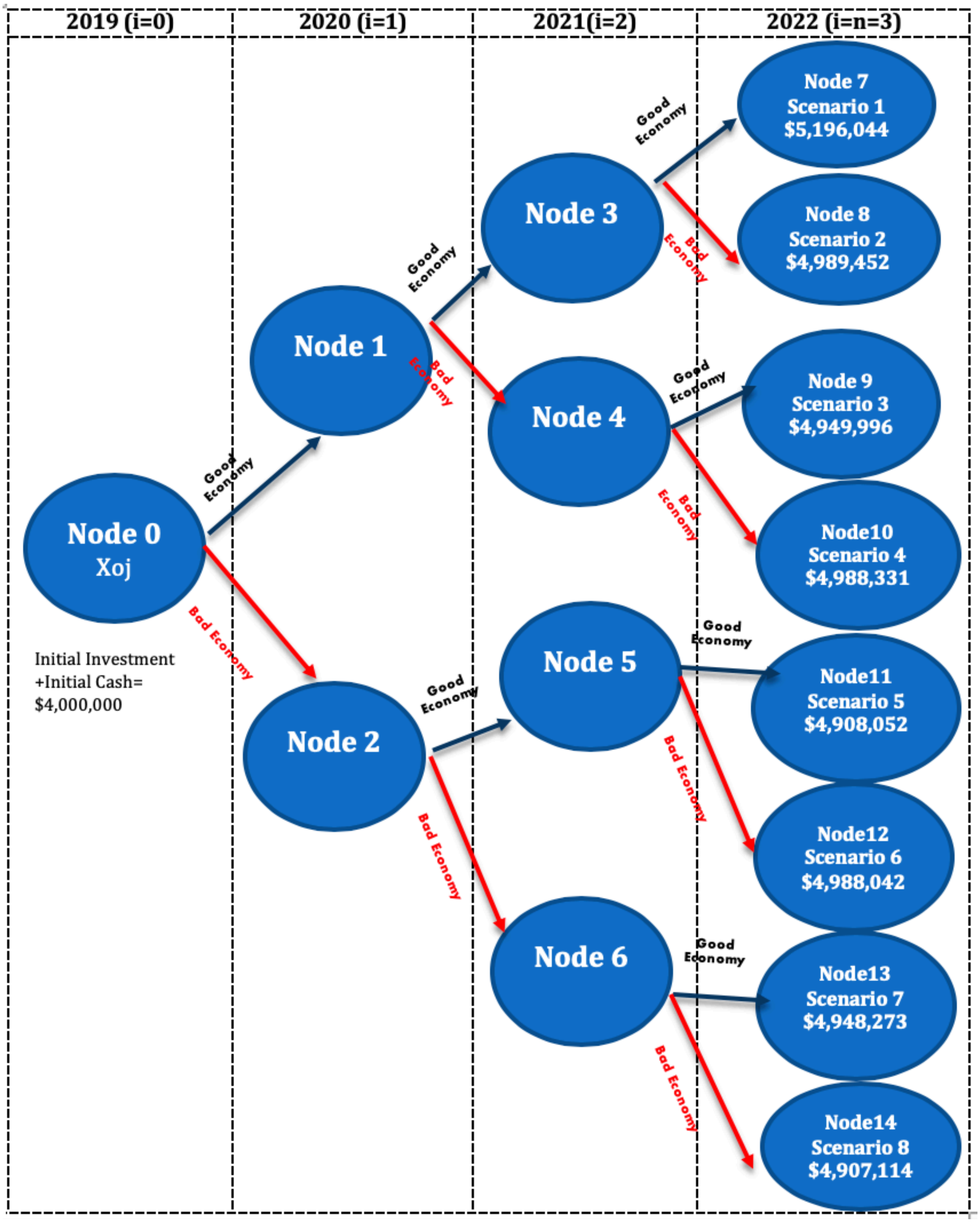
a feasible solution which can generate an objective value that slightly less than the objective value without any integer constraint.

Scenario 1 summary

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	45	5	0	49
2	ABBV	5	0	5	0	5
3	ABT	5	0	4	0	5
4	ACN	5	0	4	0	4
5	AGN	5	0	5	8	5
6	AIG	5	0	5	0	5
7	ALL	5	0	4	0	5
8	AMGN	5	0	4	0	4
9	AMZN	5	4	5	0	9
10	AXP	5	44	5	150	50
11	BA	5	0	5	0	5
12	BAC	5	0	5	0	4
13	BIIB	5	44	5	150	50
14	BK	5	0	5	150	5
15	BKNG	5	0	5	0	5
16	BLK	5	0	5	0	5
17	BMY	5	44	5	0	49
18	C	5	0	5	0	4
19	CAT	5	0	4	0	5
20	CELG	5	0	5	0	5
21	CHTR	5	0	5	0	4
22	CL	5	0	5	150	5
23	CMCSA	5	45	4	150	50
24	COF	5	0	5	0	5
25	COP	62903	0	49	0	50
26	COST	5	0	5	0	5
27	CSCO	5	0	5	0	5
28	CVS	5	0	5	0	4
29	CVX	5	45	5	0	50
30	DHR	5	0	5	0	5
31	DIS	5	0	4	0	5
32	DUK	5	0	4	0	4
33	DWDP	5	45	5	149	50
34	EMR	5	0	5	0	4
35	EXC	5	0	5	0	5
36	F	5	45	5	0	50
37	FB	5	0	5	0	5
38	FDX	5	0	5	0	5
39	FOX	5	0	5	0	4
40	FOXA	5	0	5	0	5

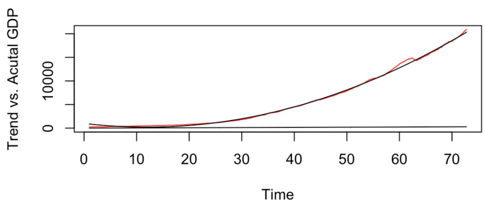
C0= $226, Objective value =$5,196,044

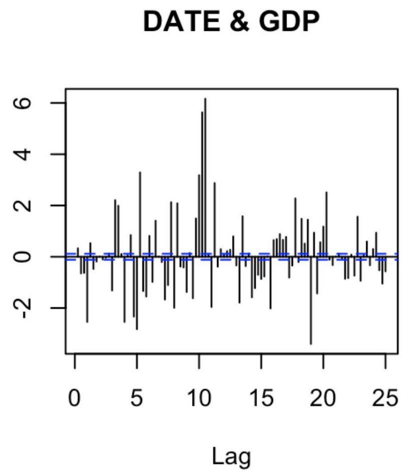
Final Result

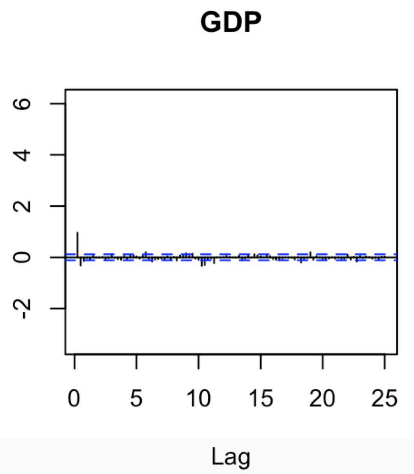


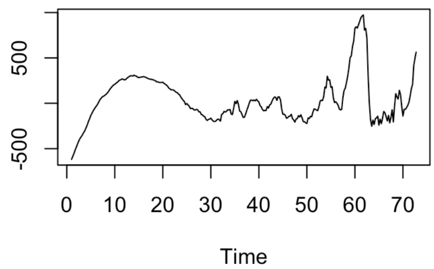
APPENDIX

Appendix 1 - Economic Conditions with Times Series









Appendix 2 - Clustering Result

Stock Name	In Good Economy	In Bad Economy	
AAPL	0	0	***Value of 1 if the performance is better, otherwise 0.***
ABBV	1	0	
ABT	0	1	
ACN	1	1	
AGN	1	1	
AIG	0	1	
ALL	1	1	
AMGN	1	1	
AMZN	1	1	
AXP	1	0	
BA	1	0	
BAC	1	1	
BIIB	0	0	
BK	1	0	
BKNG	1	0	
BLK	1	1	
BMY	1	1	
C	1	1	
CAT	1	0	
CELG	0	1	
CHTR	0	1	
CL	0	0	
CMCSA	1	0	
COF	0	0	
COP	1	0	
COST	0	0	
CSCO	0	1	
CVS	1	0	
CVX	1	0	
DHR	0	1	
DIS	0	0	
DUK	0	1	
DWDP	1	0	
EMR	0	0	
EXC	0	1	
F	0	1	
FB	0	1	
FDX	0	1	
FOX	0	0	
FOXA	1	1	

Appendix 3-Optimal Solution for Scenario 2 to 8

Scenario2:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [11]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [21]     5.00000     5.00000     5.00000     5.00000     5.00000 19429.50980     5.00000     5.00000     5.00000     5.00000
 [31]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [41]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000    45.00000     0.00000    23.88741    45.00000
 [51]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
 [61]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
 [71]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000    45.00000     0.00000     0.00000     0.00000
 [81]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [91]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
[101]     5.00000     5.00000     5.00000     5.00000     5.00000    50.00000     5.00000     5.00000     5.00000     5.00000
[111]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
[121]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000    17.60508     0.00000
[131]     0.00000     0.00000   150.00000     0.00000     0.00000     0.00000   150.00000     0.00000     0.00000     0.00000
[141]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
[151]   150.00000     0.00000     0.00000     0.00000     0.00000   150.00000     0.00000     0.00000   150.00000     0.00000
[161]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000    50.00000     5.00000    28.88741    50.00000
[171]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
[181]     5.00000     5.00000     5.00000     5.00000     5.00000    50.00000     5.00000     5.00000     5.00000     5.00000
[191]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000    50.00000     5.00000     5.00000     5.00000
[201]   200.00000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5     5 19430     5     5     5     5     5     5     5     5     5     5     5     5     5     5     0     0
 [43]     0     0     0     0    45     0    24    45     0     0     0     0     0     0     0     0     0     0     0     0     0
 [64]     0     0     0     0     0     0     0     0     0     0     0     0     0    45     0     0     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
[106]    50     5     5     5     5     5     5     5     5     5     5     5     5     5     5     0     0     0     0     0     0
[127]     0     0    18     0     0     0   150     0     0     0   150     0     0     0     0     0     0     0     0     0     0
[148]     0     0     0   150     0     0     0     0   150     0     0   150     0     5     5     5     5     5     5    50     5
[169]    29    50     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5    50     5     5     5
[190]     5     5     5     5     5     5     5    50     5     5     5   100
> #Upperbound objective value
> objup
[1] 4989432
> # Lowerbound value of objective value
> objlow
[1] 4989452
```

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	0	5	0	5
2	ABBV	5	0	5	0	5
3	ABT	5	0	4	0	4
4	ACN	5	0	5	0	4
5	AGN	5	0	4	0	4
6	AIG	5	0	4	0	4
7	ALL	5	44	4	0	50
8	AMGN	5	0	5	0	4
9	AMZN	5	23	5	17	28
10	AXP	5	45	4	0	50
11	BA	5	0	5	0	5
12	BAC	5	0	5	0	5
13	BIIB	5	0	5	150	5
14	BK	5	0	5	0	5
15	BKNG	5	0	5	0	5
16	BLK	5	0	5	0	5
17	BMY	5	0	4	150	5
18	C	5	0	5	0	4
19	CAT	5	0	5	0	4
20	CELG	5	0	5	0	5
21	CHTR	5	0	5	0	5
22	CL	5	0	5	0	5
23	CMCSA	5	0	5	0	5
24	COF	5	0	4	0	5
25	COP	5	0	4	0	5
26	COST	19430	0	50	0	50
27	CSCO	5	0	5	0	5
28	CVS	5	0	5	0	5
29	CVX	5	0	5	0	4
30	DHR	5	0	5	0	4
31	DIS	5	0	5	150	4
32	DUK	5	0	5	0	5
33	DWDP	5	0	4	0	4
34	EMR	5	0	4	0	4
35	EXC	5	0	5	0	5
36	F	5	0	5	150	4
37	FB	5	45	5	0	50
38	FDX	5	0	4	0	4
39	FOX	5	0	4	150	4
40	FOXA	5	0	5	0	4

C0= \$100, Objective value =\$4,989,452

Scenario3:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [10]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [19]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [28]     5.000000     5.000000 39631.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [37]     5.000000     5.000000     5.000000     5.000000    45.000000     0.000000     0.000000     0.000000    45.000000
 [46]     0.000000     0.000000     0.000000     8.535022     0.000000     0.000000     0.000000     0.000000     0.000000
 [55]     0.000000     0.000000    45.000000     0.000000     0.000000     0.000000     0.000000    45.000000     0.000000
 [64]     0.000000     0.000000     0.000000     0.000000     0.000000    45.000000     0.000000    45.000000     0.000000
 [73]    45.000000     0.000000    45.000000    45.000000     0.000000     0.000000     0.000000     0.000000     5.000000
 [82]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [91]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[100]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[109]     5.000000    50.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[118]     5.000000     5.000000     5.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
[127]     0.000000     0.000000    60.226817     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
[136]     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
[145]     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
[154]     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000    50.000000     5.000000
[163]     5.000000     5.000000    50.000000     5.000000     5.000000     5.000000    13.535022     5.000000     5.000000
[172]     5.000000     5.000000     5.000000     5.000000     5.000000    50.000000     5.000000     5.000000     5.000000
[181]     5.000000    50.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000    50.000000
[190]    50.000000    50.000000     5.000000    50.000000     5.000000    50.000000    50.000000     5.000000     5.000000
[199]     5.000000     5.000000   200.000000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5     5     5     5     5     5 39631     5     5     5     5     5     5     5     5     5     5    45     0
 [43]     0     0    45     0     0     0     9     0     0     0     0     0     0     0    45     0     0     0     0    45     0
 [64]     0     0     0     0     0    45     0    45     0    45     0    45    45     0     0     0     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
[106]     5     5     5     5    50     5     5     5     5     5     5     5     5     5     5     0     0     0     0     0     0
[127]     0     0    60     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0
[148]     0     0     0     0     0     0     0     0     0     0     0     0     0    50     5     5     5    50     5     5     5
[169]    14     5     5     5     5     5     5     5    50     5     5     5     5    50     5     5     5     5     5     5    50
[190]    50    50     5    50     5    50    50     5     5     5     5   200
> #Upperbound objective value
> objup
[1] 4950000
> # Lowerbound value of objective value
> objlow
[1] 4949996
```

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	45	4	0	50
2	ABBV	5	0	4	0	4
3	ABT	5	0	5	0	5
4	ACN	5	0	5	0	4
5	AGN	5	45	4	0	50
6	AIG	5	0	5	0	4
7	ALL	5	0	4	0	4
8	AMGN	5	0	5	0	4
9	AMZN	5	8	4	60	13
10	AXP	5	0	5	0	5
11	BA	5	0	5	0	4
12	BAC	5	0	5	0	5
13	BIIB	5	0	5	0	5
14	BK	5	0	5	0	5
15	BKNG	5	0	4	0	5
16	BLK	5	0	4	0	5
17	BMY	5	45	5	0	50
18	C	5	0	5	0	5
19	CAT	5	0	5	0	5
20	CELG	5	0	4	0	4
21	CHTR	5	0	5	0	5
22	CL	5	45	5	0	50
23	CMCSA	5	0	5	0	5
24	COF	5	0	5	0	5
25	COP	5	0	5	0	5
26	COST	5	0	5	0	5
27	CSCO	5	0	5	0	5
28	CVS	5	0	5	0	5
29	CVX	5	45	5	0	50
30	DHR	39631	0	50	0	50
31	DIS	5	44	5	0	50
32	DUK	5	0	5	0	5
33	DWDP	5	45	4	0	50
34	EMR	5	0	4	0	4
35	EXC	5	44	5	0	50
36	F	5	44	5	0	50
37	FB	5	0	4	0	4
38	FDX	5	0	5	0	4
39	FOX	5	0	5	0	5
40	FOXA	5	0	5	0	4

C0= $200, Objective value =$4,949,996

Scenario4:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [11]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [21]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [31]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000 82559.16667
 [41]     0.00000     0.00000     0.00000     0.00000    45.00000     0.00000     0.00000     0.00000     0.00000    45.00000
 [51]     0.00000     0.00000    44.64621     0.00000     0.00000     0.00000    45.00000     0.00000     0.00000     0.00000
 [61]     0.00000    45.00000     0.00000     0.00000     0.00000     0.00000    45.00000     0.00000     0.00000     0.00000
 [71]    45.00000     0.00000     0.00000     0.00000     0.00000    45.00000    45.00000     0.00000    45.00000     0.00000
 [81]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
 [91]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
[101]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000
[111]     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000     5.00000    50.00000
[121]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000   150.00000
[131]     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
[141]     0.00000   150.00000     0.00000     0.00000     0.00000     0.00000   150.00000     0.00000     0.00000     0.00000
[151]     0.00000   150.00000     0.00000   150.00000   150.00000     0.00000     0.00000   150.00000   110.11366     0.00000
[161]     5.00000     5.00000     5.00000     5.00000    50.00000     5.00000     5.00000     5.00000     5.00000    50.00000
[171]     5.00000     5.00000    49.64621     5.00000     5.00000     5.00000    50.00000     5.00000     5.00000     5.00000
[181]     5.00000    50.00000     5.00000     5.00000     5.00000     5.00000    50.00000     5.00000     5.00000     5.00000
[191]    50.00000     5.00000     5.00000     5.00000     5.00000    50.00000    50.00000     5.00000    50.00000    50.00000
[201]   200.00000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5 82559     0     0
 [43]     0     0    45     0     0     0     0    45     0     0    45     0     0     0    45     0     0     0     0    45     0
 [64]     0     0     0    45     0     0     0    45     0     0     0     0    45    45     0    45     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
[106]     5     5     5     5     5     5     5     5     5     5     5     5     5     5    50     0     0     0     0     0     0
[127]     0     0     0   150     0     0     0     0     0     0     0     0     0     0     0   150     0     0     0     0   150
[148]     0     0     0     0   150     0   150   150     0     0   150   110     0     5     5     5     5    50     5     5     5
[169]     5    50     5     5    50     5     5     5    50     5     5     5     5    50     5     5     5     5    50     5     5
[190]     5    50     5     5     5     5    50    50     5    50    50   208
> #Upperbound objective value
> objup
[1] 4988336
> # Lowerbound value of objective value
> objlow
[1] 4988331
```

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	0	5	0	5
2	ABBV	5	0	5	0	5
3	ABT	5	0	5	0	5
4	ACN	5	0	5	0	5
5	AGN	5	45	5	0	50
6	AIG	5	0	5	0	5
7	ALL	5	0	5	0	5
8	AMGN	5	0	5	0	4
9	AMZN	5	0	5	0	4
10	AXP	5	44	5	149	50
11	BA	5	0	5	0	5
12	BAC	5	0	5	0	5
13	BIIB	5	44	5	0	49
14	BK	5	0	5	0	4
15	BKNG	5	0	5	0	4
16	BLK	5	0	5	0	5
17	BMY	5	45	5	0	50
18	C	5	0	5	0	5
19	CAT	5	0	4	0	4
20	CELG	5	0	5	0	4
21	CHTR	5	0	5	0	5
22	CL	5	44	5	150	50
23	CMCSA	5	0	5	0	4
24	COF	5	0	5	0	5
25	COP	5	0	5	0	5
26	COST	5	0	5	0	4
27	CSCO	5	45	5	150	49
28	CVS	5	0	5	0	5
29	CVX	5	0	5	0	4
30	DHR	5	0	5	0	4
31	DIS	5	45	5	0	50
32	DUK	5	0	5	150	5
33	DWDP	5	0	5	0	5
34	EMR	5	0	4	149	4
35	EXC	5	0	5	150	5
36	F	5	45	5	0	50
37	FB	5	45	4	0	50
38	FDX	5	0	5	150	5
39	FOX	5	44	5	110	50
40	FOXA	82559	0	50	0	50

C0= $208, Objective value =$4,988,331

Scenario5:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [11]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [21]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [31]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000 82559.16667
 [41]     45.00000      0.00000      0.00000      0.00000     45.00000      0.00000      0.00000      0.00000      0.00000     45.00000
 [51]      0.00000      0.00000      0.00000      0.00000      0.00000     17.75432      0.00000      0.00000      0.00000      0.00000
 [61]      0.00000     45.00000      0.00000      0.00000      0.00000      0.00000     45.00000      0.00000     45.00000      0.00000
 [71]      0.00000      0.00000     45.00000      0.00000     45.00000     45.00000      0.00000      0.00000     45.00000      0.00000
 [81]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [91]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[101]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[111]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000     50.00000
[121]    150.00000      0.00000      0.00000      0.00000    150.00000      0.00000      0.00000      0.00000      0.00000    150.00000
[131]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
[141]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000    150.00000      0.00000      0.00000      0.00000
[151]      0.00000      0.00000    150.00000      0.00000    150.00000    150.00000     57.55079      0.00000      0.00000      0.00000
[161]     50.00000      5.00000      5.00000      5.00000     50.00000      5.00000      5.00000      5.00000      5.00000     50.00000
[171]      5.00000      5.00000      5.00000      5.00000      5.00000     22.75432      5.00000      5.00000      5.00000      5.00000
[181]      5.00000     50.00000      5.00000      5.00000      5.00000      5.00000     50.00000      5.00000     50.00000      5.00000
[191]      5.00000      5.00000     50.00000      5.00000     50.00000     50.00000      5.00000      5.00000     50.00000     50.00000
[201]    200.00000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5 82559    45     0
 [43]     0     0    45     0     0     0     0    45     0     0     0     0     0    18     0     0     0     0     0    45     0
 [64]     0     0     0    45     0    45     0     0     0    45     0    45    45     0     0    45     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
[106]     5     5     5     5     5     5     5     5     5     5     5     5     5     5    50   150     0     0     0   150     0
[127]     0     0     0   150     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0   150
[148]     0     0     0     0     0   150     0   150   150    58     0     0     0    50     5     5     5    50     5     5     5
[169]     5    50     5     5     5     5     5    23     5     5     5     5     5    50     5     5     5     5    50     5    50
[190]     5     5     5    50     5    50    50     5     5    50    50   208
> #Upperbound objective value
> objup
[1] 4908053
> # Lowerbound value of objective value
> objlow
[1] 4908052
```

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	45	4	150	50
2	ABBV	5	0	5	0	4
3	ABT	5	0	5	0	5
4	ACN	5	0	4	0	5
5	AGN	5	45	5	150	50
6	AIG	5	0	5	0	4
7	ALL	5	0	4	0	4
8	AMGN	5	0	4	0	5
9	AMZN	5	0	5	0	4
10	AXP	5	45	5	150	49
11	BA	5	0	5	0	5
12	BAC	5	0	4	0	5
13	BIIB	5	0	5	0	5
14	BK	5	0	4	0	5
15	BKNG	5	0	5	0	4
16	BLK	5	17	5	0	22
17	BMY	5	0	5	0	4
18	C	5	0	5	0	5
19	CAT	5	0	5	0	4
20	CELG	5	0	5	0	4
21	CHTR	5	0	5	0	5
22	CL	5	45	4	0	50
23	CMCSA	5	0	5	0	5
24	COF	5	0	5	0	5
25	COP	5	0	5	0	4
26	COST	5	0	4	0	4
27	CSCO	5	45	4	150	50
28	CVS	5	0	4	0	5
29	CVX	5	45	5	0	50
30	DHR	5	0	5	0	5
31	DIS	5	0	5	0	5
32	DUK	5	0	5	0	5
33	DWDP	5	44	5	150	50
34	EMR	5	0	5	0	4
35	EXC	5	45	4	150	50
36	F	5	45	4	149	49
37	FB	5	0	4	57	4
38	FDX	5	0	5	0	4
39	FOX	5	44	5	0	50
40	FOXA	82559	0	50	0	50

C0= $208, Objective value =$4,908,052

Scenario6:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [11]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [21]      5.00000      5.00000      5.00000      5.00000      5.00000 19429.50980      5.00000      5.00000      5.00000      5.00000
 [31]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [41]      0.00000      0.00000      0.00000      0.00000     45.00000      0.00000      0.00000      0.00000     20.40812     45.00000
 [51]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
 [61]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000     45.00000      0.00000     45.00000      0.00000
 [71]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
 [81]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [91]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[101]      5.00000      5.00000      5.00000      5.00000      5.00000     50.00000      5.00000      5.00000      5.00000      5.00000
[111]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[121]      0.00000      0.00000      0.00000      0.00000    150.00000      0.00000      0.00000      0.00000     54.89476      0.00000
[131]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
[141]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
[151]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
[161]      5.00000      5.00000      5.00000      5.00000     50.00000      5.00000      5.00000      5.00000     25.40812     50.00000
[171]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[181]      5.00000      5.00000      5.00000      5.00000      5.00000     50.00000     50.00000      5.00000     50.00000      5.00000
[191]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[201]    200.00000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5     5 19430     5     5     5     5     5     5     5     5     5     5     5     5     5     5     0     0
 [43]     0     0    45     0     0     0    20    45     0     0     0     0     0     0     0     0     0     0     0     0     0
 [64]     0     0     0    45     0    45     0     0     0     0     0     0     0     0     0     0     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
[106]    50     5     5     5     5     5     5     5     5     5     5     5     5     5     5     0     0     0     0   150     0
[127]     0     0    55     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0
[148]     0     0     0     0     0     0     0     0     0     0     0     0     0     5     5     5     5    50     5     5     5
[169]    25    50     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5    50    50     5    50
[190]     5     5     5     5     5     5     5     5     5     5     5   100
> #Upperbound objective value
> objup
[1] 4988021
> # Lowerbound value of objective value
> objlow
[1] 4988042
```

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	0	4	0	5
2	ABBV	5	0	5	0	5
3	ABT	5	0	5	0	4
4	ACN	5	0	4	0	5
5	AGN	5	45	4	150	50
6	AIG	5	0	5	0	5
7	ALL	5	0	5	0	4
8	AMGN	5	0	5	0	5
9	AMZN	5	20	5	54	25
10	AXP	5	45	5	0	50
11	BA	5	0	5	0	4
12	BAC	5	0	5	0	5
13	BIIB	5	0	4	0	4
14	BK	5	0	5	0	4
15	BKNG	5	0	5	0	4
16	BLK	5	0	4	0	5
17	BMY	5	0	5	0	5
18	C	5	0	5	0	5
19	CAT	5	0	4	0	5
20	CELG	5	0	5	0	4
21	CHTR	5	0	5	0	4
22	CL	5	0	4	0	4
23	CMCSA	5	0	4	0	4
24	COF	5	0	4	0	5
25	COP	5	0	5	0	4
26	COST	19430	0	50	0	50
27	CSCO	5	45	5	0	50
28	CVS	5	0	5	0	5
29	CVX	5	44	4	0	50
30	DHR	5	0	5	0	5
31	DIS	5	0	4	0	5
32	DUK	5	0	4	0	4
33	DWDP	5	0	5	0	4
34	EMR	5	0	4	0	5
35	EXC	5	0	5	0	4
36	F	5	0	5	0	5
37	FB	5	0	5	0	4
38	FDX	5	0	4	0	4
39	FOX	5	0	4	0	5
40	FOXA	5	0	4	0	4

C0= $100, Objective value =$4,988,042

Scenario7:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [10]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [19]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [28]     5.000000     5.000000 39631.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [37]     5.000000     5.000000     5.000000     5.000000     0.000000     0.000000     0.000000     0.000000     0.000000
 [46]     0.000000     0.000000     0.000000    21.879537     0.000000     0.000000     0.000000     0.000000     0.000000
 [55]     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
 [64]    45.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
 [73]     0.000000     0.000000     0.000000     0.000000     0.000000    45.000000     0.000000     0.000000     5.000000
 [82]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
 [91]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[100]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[109]     5.000000    50.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[118]     5.000000     5.000000     5.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
[127]   150.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000     2.368084
[136]     0.000000     0.000000     0.000000     0.000000     0.000000     0.000000   150.000000     0.000000   150.000000
[145]     0.000000     0.000000   150.000000     0.000000   150.000000     0.000000     0.000000     0.000000     0.000000
[154]     0.000000   150.000000     0.000000     0.000000   150.000000     0.000000     0.000000     5.000000     5.000000
[163]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000    26.879537     5.000000     5.000000
[172]     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[181]     5.000000     5.000000     5.000000    50.000000     5.000000     5.000000     5.000000     5.000000     5.000000
[190]    50.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000     5.000000    50.000000
[199]     5.000000     5.000000   200.000000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5     5     5     5     5     5 39631     5     5     5     5     5     5     5     5     5     5     0     0
 [43]     0     0     0     0     0     0    22     0     0     0     0     0     0     0     0     0     0     0     0     0     0
 [64]    45     0     0     0     0     0     0     0     0     0     0     0     0     0    45     0     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
[106]     5     5     5     5    50     5     5     5     5     5     5     5     5     5     5     0     0     0     0     0     0
[127]   150     0     0     0     0     0     0     0     2     0     0     0     0     0     0   150     0   150     0     0   150
[148]     0   150     0     0     0     0     0   150     0     0   150     0     0     5     5     5     5     5     5     5     5
[169]    27     5     5     5     5     5     5     5     5     5     5     5     5     5     5    50     5     5     5     5     5
[190]    50     5     5     5     5     5     5     5    50     5     5   200
> #Upperbound objective value
> objup
[1] 4948283
> # Lowerbound value of objective value
> objlow
[1] 4948273
```

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	0	5	0	4
2	ABBV	5	0	5	0	5
3	ABT	5	0	5	0	5
4	ACN	5	0	5	0	5
5	AGN	5	0	5	0	5
6	AIG	5	0	4	0	5
7	ALL	5	0	5	150	5
8	AMGN	5	0	5	0	5
9	AMZN	5	21	4	0	26
10	AXP	5	0	4	0	4
11	BA	5	0	4	0	4
12	BAC	5	0	4	0	4
13	BIIB	5	0	4	0	5
14	BK	5	0	4	0	4
15	BKNG	5	0	4	2	4
16	BLK	5	0	4	0	5
17	BMY	5	0	5	0	5
18	C	5	0	5	0	5
19	CAT	5	0	5	0	5
20	CELG	5	0	4	0	4
21	CHTR	5	0	5	0	5
22	CL	5	0	5	150	5
23	CMCSA	5	0	4	0	5
24	COF	5	45	4	150	50
25	COP	5	0	5	0	4
26	COST	5	0	5	0	5
27	CSCO	5	0	5	150	5
28	CVS	5	0	5	0	5
29	CVX	5	0	4	150	4
30	DHR	39631	0	50	0	50
31	DIS	5	0	4	0	4
32	DUK	5	0	4	0	5
33	DWDP	5	0	4	0	5
34	EMR	5	0	4	0	4
35	EXC	5	0	4	150	5
36	F	5	0	5	0	4
37	FB	5	0	4	0	5
38	FDX	5	45	4	150	50
39	FOX	5	0	5	0	5
40	FOXA	5	0	4	0	4

C0= $208, Objective value =$4,948,273

Scenario8:

```
> #### Solution ####
> #Upperbound value of decision variable
> upsol
  [1]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [11]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [21]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000 39631.00000
 [31]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [41]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000     15.58868      0.00000
 [51]      0.00000      0.00000      0.00000      0.00000      0.00000     45.00000      0.00000      0.00000      0.00000      0.00000
 [61]      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
 [71]      0.00000      0.00000     45.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
 [81]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
 [91]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[101]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000     50.00000
[111]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[121]      0.00000      0.00000    150.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
[131]      0.00000      0.00000      0.00000      0.00000      0.00000     62.01410      0.00000      0.00000      0.00000      0.00000
[141]      0.00000    150.00000      0.00000      0.00000      0.00000      0.00000    150.00000      0.00000    150.00000      0.00000
[151]      0.00000      0.00000    150.00000    150.00000      0.00000      0.00000      0.00000      0.00000    150.00000      0.00000
[161]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000     20.58868      5.00000
[171]      5.00000      5.00000      5.00000      5.00000      5.00000     50.00000      5.00000      5.00000      5.00000      5.00000
[181]      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000     50.00000
[191]      5.00000      5.00000     50.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000      5.00000
[201]    200.00000
> # Lowerbound value of decision variable
> lowsol
  [1]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
 [22]     5     5     5     5     5     5     5     5 39631     5     5     5     5     5     5     5     5     5     5     0     0
 [43]     0     0     0     0     0     0    16     0     0     0     0     0     0    45     0     0     0     0     0     0     0
 [64]     0     0     0     0     0     0     0     0     0    45     0     0     0     0     0     0     0     5     5     5     5
 [85]     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5     5
[106]     5     5     5     5    50     5     5     5     5     5     5     5     5     5     5     0     0   150     0     0     0
[127]     0     0     0     0     0     0     0     0     0    62     0     0     0     0     0   150     0     0     0     0   150
[148]     0   150     0     0     0   150   150     0     0     0     0   150     0     5     5     5     5     5     5     5     5
[169]    21     5     5     5     5     5     5    50     5     5     5     5     5     5     5     5     5     5     5     5     5
[190]    50     5     5    50     5     5     5     5     5     5     5   200
> #Upperbound objective value
> objup
[1] 4907121
> # Lowerbound value of objective value
> objlow
[1] 4907114
```

Index	Stock	x0	# Shares buy period 1	# Shares sell period 1	# Shares buy period 2	# Shares sell period 2
1	AAPL	5	0	5	0	5
2	ABBV	5	0	5	0	4
3	ABT	5	0	5	150	5
4	ACN	5	0	5	0	4
5	AGN	5	0	5	0	5
6	AIG	5	0	5	0	4
7	ALL	5	0	5	0	4
8	AMGN	5	0	5	0	4
9	AMZN	5	15	5	0	20
10	AXP	5	0	5	0	5
11	BA	5	0	5	0	5
12	BAC	5	0	5	0	5
13	BIIB	5	0	5	0	5
14	BK	5	0	4	0	5
15	BKNG	5	0	4	0	4
16	BLK	5	44	5	62	50
17	BMY	5	0	5	0	5
18	C	5	0	4	0	5
19	CAT	5	0	4	0	5
20	CELG	5	0	5	0	5
21	CHTR	5	0	4	0	5
22	CL	5	0	4	150	4
23	CMCSA	5	0	5	0	5
24	COF	5	0	4	0	4
25	COP	5	0	4	0	4
26	COST	5	0	5	0	5
27	CSCO	5	0	4	149	4
28	CVS	5	0	5	0	5
29	CVX	5	0	5	150	5
30	DHR	39631	0	50	0	50
31	DIS	5	0	5	0	5
32	DUK	5	0	5	0	4
33	DWDP	5	44	5	150	50
34	EMR	5	0	4	150	4
35	EXC	5	0	4	0	4
36	F	5	0	5	0	5
37	FB	5	0	5	0	4
38	FDX	5	0	4	0	5
39	FOX	5	0	4	150	5
40	FOXA	5	0	5	0	5

C0= $200, Objective value =$4,907,114